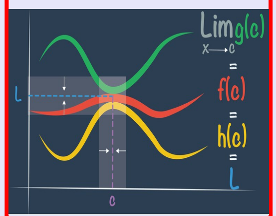
Math 261

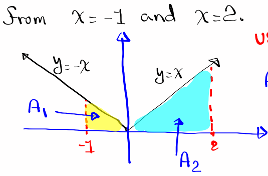
Fall 2022

Lecture 38



Feb 19-8:47 AM

Find the area below $f(x) = |x|$, above x-axis from $x=-1$ and $x=2$.



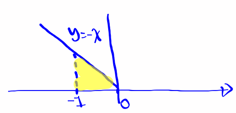
using geometry

$A_1 = \frac{b \cdot h}{2} = \frac{1 \cdot 1}{2} = \frac{1}{2}$ ✓

$A_2 = \frac{b \cdot h}{2} = \frac{2 \cdot 2}{2} = 2$

$\boxed{\text{Total Area} = 2.5}$

Now using Calc.



$\Delta x = \frac{b-a}{n} = \frac{0-(-1)}{n} = \frac{1}{n}$

$x_i = a + i\Delta x = -1 + \frac{i}{n}$

$f(x_i) = -\left(-1 + \frac{i}{n}\right) = 1 - \frac{i}{n}$

$A_i = \Delta x \cdot f(x_i) = \frac{1}{n} \cdot \left(1 - \frac{i}{n}\right)$

Yellow Area $= \lim_{n\to\infty} \sum_{i=1}^{n} A_i = \lim_{n\to\infty} \sum_{i=1}^{n} \frac{1}{n}\left(1 - \frac{i}{n}\right)$

$\sum_{i=1}^{n} i = \frac{n(n+1)}{2}$

$= \lim_{n\to\infty} \frac{1}{n}\left[\sum_{i=1}^{n} 1 - \sum_{i=1}^{n} \frac{i}{n}\right]$

$= \lim_{n\to\infty} \frac{1}{n}\left[n - \frac{1}{n}\sum_{i=1}^{n} i\right]$

$= \lim_{n\to\infty} \frac{1}{n}\left[n - \frac{1}{n} \cdot \frac{n(n+1)}{2}\right]$

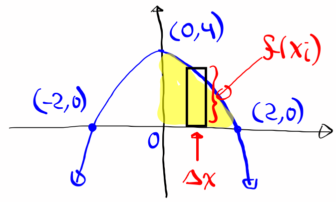
Make sure to do the blue area.

$= \lim_{n\to\infty} \left[1 - \frac{n^2+n}{2n^2}\right]$

$= 1 - \frac{1}{2} = \boxed{\frac{1}{2}}$

Nov 3-8:49 AM

Find the area of the enclosed region by $f(x) = 4 - x^2$ and the x-axis.



$\Delta x = \frac{b-a}{n} = \frac{2-0}{n} = \frac{2}{n}$

$x_i = a + i\,\Delta x$
$= 0 + i \cdot \frac{2}{n} = \frac{2i}{n}$

$f(x_i) = 4 - \left(\frac{2i}{n}\right)^2$

$A_i = \Delta x \cdot f(x_i)$
$= \frac{2}{n} \cdot \left[4 - \frac{4i^2}{n^2}\right]$

Final Area

$= 2 \lim_{n \to \infty} \sum_{i=1}^{n} A_i$

$= 2 \lim_{n \to \infty} \sum_{i=1}^{n} \frac{2}{n}\left[4 - \frac{4i^2}{n^2}\right] = 2 \lim_{n \to \infty} \frac{2}{n}\left[4n - \frac{4}{n^2} \cdot \sum_{i=1}^{n} i^2\right]$

$= 2 \lim_{n \to \infty} \frac{2}{n}\left[4n - \frac{4}{n^2} \cdot \frac{n(n+1)(2n+1)}{6}\right]$ $\quad \frac{16}{6} = \frac{8}{3}$

$= 2 \lim_{n \to \infty} \left[8 - \frac{16n^3 + \text{Junk}}{6n^3}\right]$

$= 2\left(8 - \frac{16}{6}\right) = 16 - \frac{16}{3} = \boxed{\frac{32}{3}}$

Nov 3-9:02 AM

Intro. to integrals

Definite integral $\Rightarrow$ final answer is a value

$$\int_a^b f(x)\,dx = \lim_{n \to \infty} \sum_{i=1}^{n} f(x_i) \cdot \Delta x$$

integral

Assuming this limit exists.

Now $\int f(x)\,dx = F(x) + C$ where $F'(x) = f(x)$

Integrand is the first derivative of answer

$\int x\,dx = \frac{x^2}{2} + C$

$\int (4 - x^2)\,dx = 4x - \frac{x^3}{3} + C$

$\int (\sec^2 x + \cos x)\,dx = \tan x + \sin x + C$

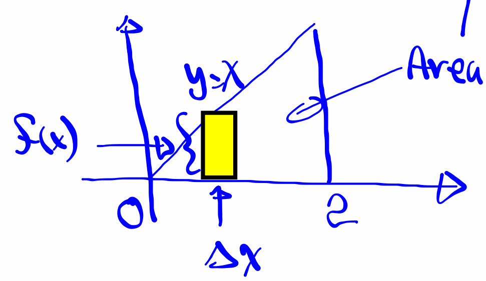
Nov 3-9:16 AM

Definite integral

$$\int_a^b f(x)\,dx = F(x)\Big|_a^b = F(b) - F(a)$$

$$F'(x) = f(x)$$

$$\int_0^2 x\,dx = \frac{x^2}{2}\Big|_0^2 = \frac{2^2}{2} - \frac{0^2}{2} = \frac{4}{2} - 0 = \boxed{2}$$



Nov 3-9:24 AM

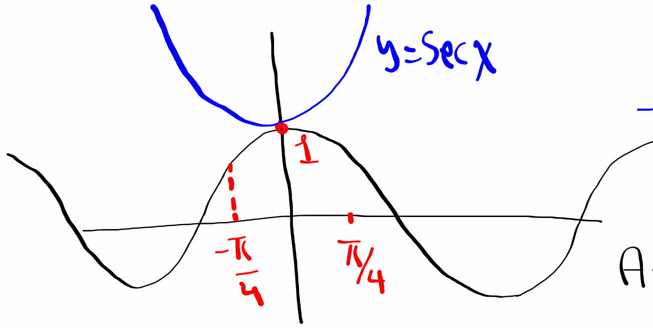
Evaluate $\displaystyle\int_0^2 (4 - x^2)\,dx = \left(4x - \frac{x^3}{3}\right)\Big|_0^2$

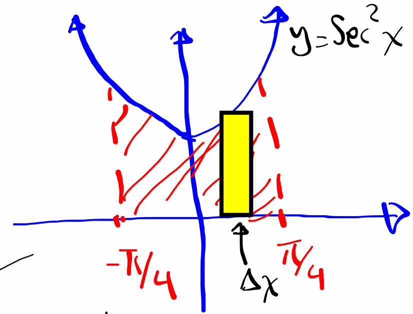
$$= \left(4 \cdot 2 - \frac{2^3}{3}\right) - \left(4 \cdot 0 - \frac{0^3}{3}\right)$$

$$= 8 - \frac{8}{3} = \boxed{\frac{16}{3}}$$

Nov 3-9:29 AM

Find the area below $f(x) = \sec^2 x$, above x-axis from $x = \frac{-\pi}{4}$ to $x = \frac{\pi}{4}$.





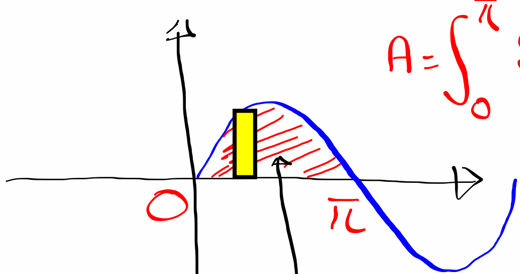
$$A = \int_{-\pi/4}^{\pi/4} \sec^2 x \, dx$$

$$A = 2\int_0^{\pi/4} \sec^2 x \, dx = 2\tan x \Big|_0^{\pi/4} = 2\left[\tan\frac{\pi}{4} - \tan 0\right]$$

$$= 2(1 - 0) = \boxed{2}$$

Nov 3-9:33 AM

Find the area enclosed by $y = \sin x$, x-axis, in QI for $0 \le x \le \pi$.



$$A = \int_0^{\pi} \sin x \, dx = -\cos x \Big|_0^{\pi}$$

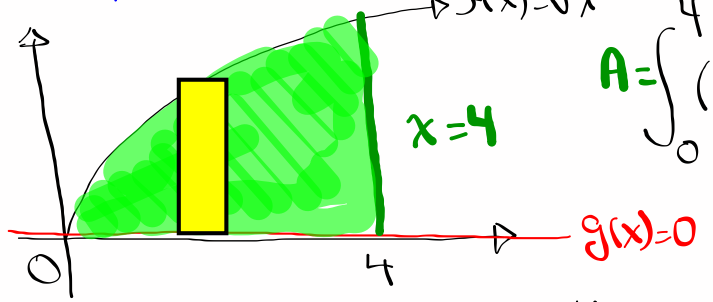
$$= -\left[\cos\pi - \cos 0\right]$$

$$= -\left[-1 - 1\right]$$

$$= -(-2) = \boxed{2}$$

Nov 3-9:41 AM

Find the area enclosed by $f(x)=\sqrt{x}$, $g(x)=0$, and $x=4$.



$$A=\int_0^4 (\sqrt{x}-0)\,dx$$

Top

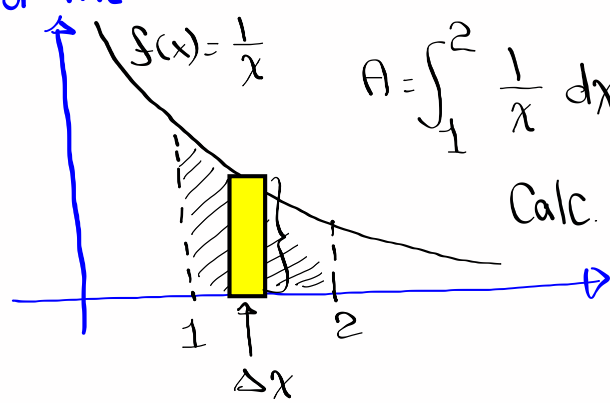
Bottom

$$A=\int_0^4 \sqrt{x}\,dx=\int_0^4 x^{1/2}dx=\frac{x^{3/2}}{3/2}\Big|_0^4=\frac{2x\sqrt{x}}{3}\Big|_0^4$$

$$=\frac{2}{3}\cdot 4\sqrt{4}=\boxed{\frac{16}{3}}$$

Nov 3-9:46 AM

Find the shaded area below



$$A=\int_1^2 \frac{1}{x}\,dx$$

Calc. II

Nov 3-9:52 AM